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LAMSS
Laboratory for Active Materials and Smart Structures

A novel Helmholtz potential approach to predicting acoustic guided waves generated by fatigue crack

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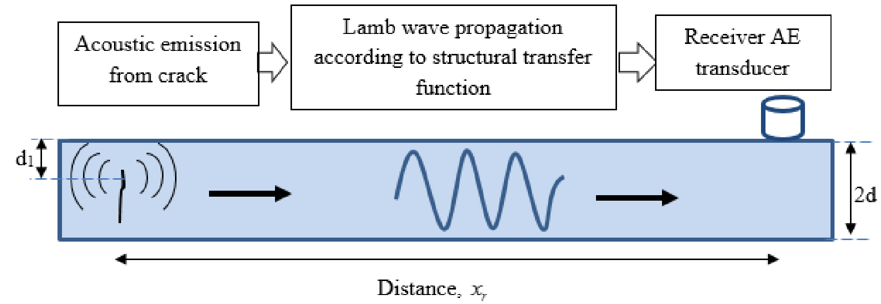
Outline

- Motivation
- Helmholtz potential approach to the analysis of acoustic emission (AE) guided wave
  - Theoretical formulation
  - Developing forward problem
- Developing inverse algorithm
- Experiments
- Summary, conclusions and future work

Objective

- Acoustic Emission (AE) due to released energy
  - Non-destructive testing (NDT)
  - Locating and monitoring crack/ damage
  - Crack characterization
- Applications:



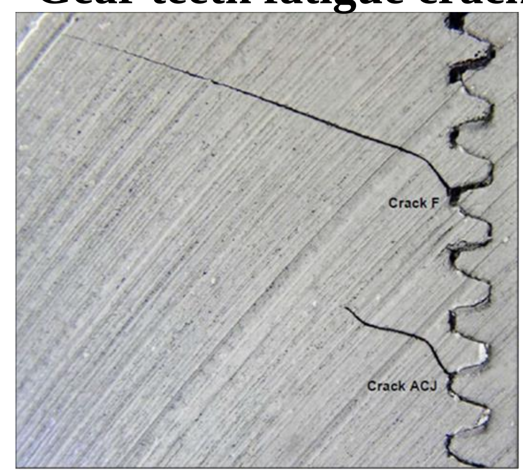
Nuclear spent fuel tank

Ref: http://www.sseb.org/downloads/Presentations/

Aircraft fuselage

Ref: FSIMS Document

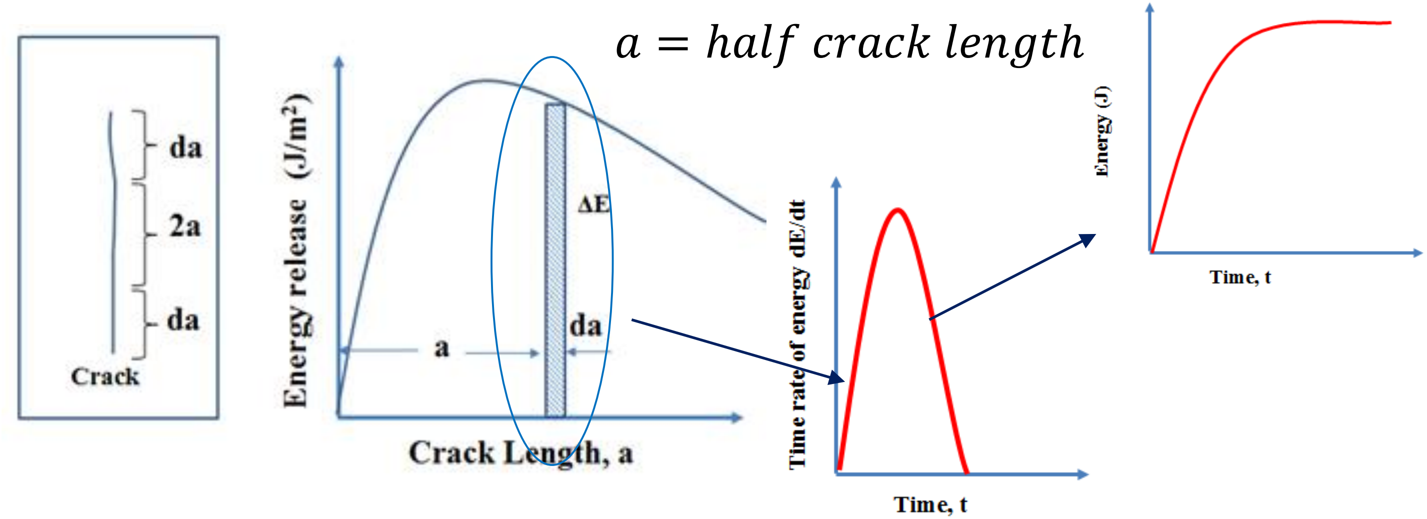
Gear teeth fatigue crack

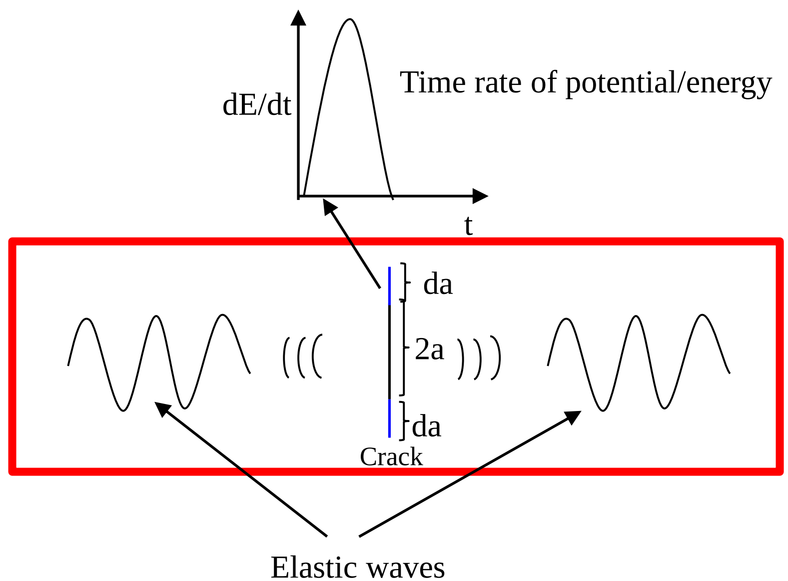


Ref: http://http://www.twi-global.com

Overview of Potential Approach

$a = half\ crack\ length$





Elastodynamic (Navier-Lame) Equations

- Navier-Lame equations in vector form

$$(\lambda + \mu)\vec{\nabla}\left(\vec{\nabla} \cdot \vec{u}\right) + \mu\nabla^2\vec{u} = \rho\ddot{\vec{u}}$$

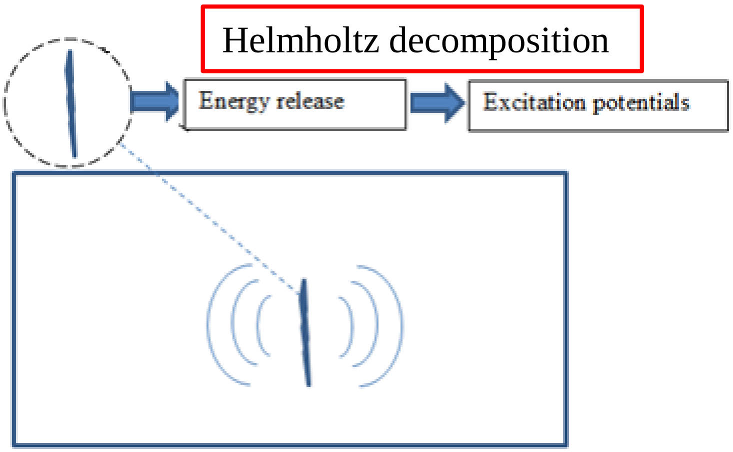
$$\vec{u} = u_x\hat{i} + u_y\hat{j} + u_z\hat{k}$$

Lame constant λ, μ

Density, ρ

- The basic concepts and equations of elastodynamics have been explained by
  - Lamb (1917)
  - Viktorov (1967)
  - Graff (1991)
  - Aki and Richards (2002)
  - Achenbach (2003)
  - Giurgiutiu (2014) etc.

Formation Pressure and Shear Excitation Potentials



Formation of scalar and vector excitation potentials

- If body force is present, then the Navier-Lame equation can be written as follow

$$(\lambda+\mu)\vec{\nabla}\left(\vec{\nabla}\cdot\vec{u}\right)+\mu\nabla^2\vec{u}+\rho\vec{f}=\rho\ddot{\vec{u}}$$

$\vec{f}=f_x\hat{i}+f_y\hat{j}+f_z\hat{k}$

$\vec{u}=u_x\hat{i}+u_y\hat{j}+u_z\hat{k}$

Lame constant λ,μ

Density, ρ

- Helmholtz decomposition states that any vector can be resolved into two potentials

$$\vec{u}=grad\ \Phi+curl\ \vec{H}=\vec{\nabla}\Phi+\vec{\nabla}\times\vec{H}$$

$$\vec{f}=grad\ A^*+curl\ \vec{B}^*=\vec{\nabla}A^*+\vec{\nabla}\times\vec{B}^*$$

$\vec{\nabla}\cdot\vec{H}=0$ $\qquad$ $\vec{H}=H_x\vec{i}+H_y\vec{j}+H_z\vec{k}$

$\vec{\nabla}\cdot\vec{B}^*=0$ $\qquad$ $\vec{B}^*=B_x^*\vec{i}+B_y^*\vec{j}+B_z^*\vec{k}$

Wave Equation for Potentials

- Inhomogeneous wave equation for potentials

$$c_P^2 \nabla^2 \Phi + A^* = \ddot{\Phi}$$

$$c_S^2 \nabla^2 \vec{H} + \vec{B}^* = \ddot{\vec{H}} \longrightarrow \begin{cases} c_S^2 \nabla^2 H_x + B_x^* = \ddot{H}_x \\ c_S^2 \nabla^2 H_y + B_y^* = \ddot{H}_y \\ c_S^2 \nabla^2 H_z + B_z^* = \ddot{H}_z \end{cases}$$

- So for P+SV waves the relevant potentials are

$$\Phi, H_z, A^*, B_z^*$$

- Corresponding wave equations for potentials are

$$c_P^2 \nabla^2 \Phi + A^* = \ddot{\Phi}$$
$$c_S^2 \nabla^2 H_z + B_z^* = \ddot{H}_z$$

P+SV waves

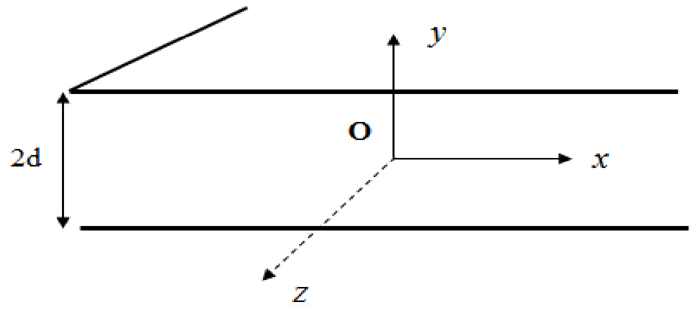
Unknown displacement potentials (pressure and shear)

Excitation potentials (pressure and shear)

- Equations must be solved subject to zero-stress boundary conditions

Concentrated Potentials

$$\sigma_{xx}\Big|_{y=\pm d}=0,\ \sigma_{yy}\Big|_{y=\pm d}=0,\ \sigma_{xy}\Big|_{y=\pm d}=0$$



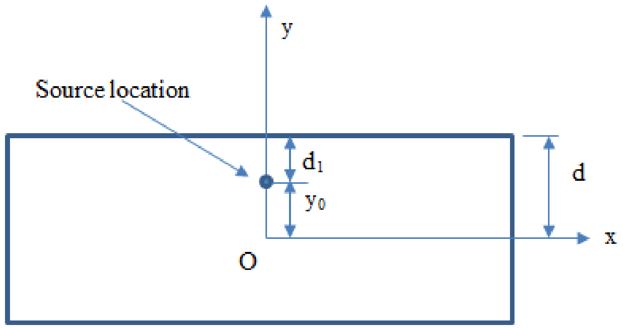


Plate of thickness 2d in which straight crested Lamb waves propagate in the x direction due to concentrated potential at $x=0; y=0$

- The potentials due to force can be written as follow

$$A = A\delta(x)\delta(y-y_0)e^{-i\omega t} \qquad B_z = B_z\delta(x)\delta(y-y_0)e^{-i\omega t}$$

$$c_P^2\nabla^2\Phi + \mathrm{A} = \ddot{\Phi}$$
$$c_S^2\nabla^2\vec{H}_z + \vec{B}_z = \ddot{\vec{H}}_z$$

- Wave equations for potentials become

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}+\frac{\omega^2}{c_P^2}\Phi = -A\delta(x)\delta(y-y_0)$$

$$\frac{\partial^2 H_z}{\partial x^2}+\frac{\partial^2 H_z}{\partial y^2}+\frac{\omega^2}{c_S^2}H_z = -B_z\delta(x)\delta(y-y_0)$$

In-plane Strain Solution

Wave equations for potentials in a plate

↓

Fourier transform in x direction

↓

Thickness dependent differential equation

↓

Total solution:
(a) General solution for homogeneous equation
(b) Particular solution due to source

↓

Symmetric and antisymmetric Lamb wave solution

↓

In-plane strain displacement is calculated

P+SV waves

$$c_P^2\nabla^2\Phi + A = \ddot{\Phi}$$
$$c_S^2\nabla^2\vec{H}_z + \vec{B}_z = \ddot{\vec{H}}_z$$

$$\bar{\Phi}''(\xi,y) + \eta_p^2\bar{\Phi}(\xi,y) = -A\delta(y-y_0)$$
$$\bar{H}_z''(\xi,y) + \eta_s^2\bar{H}_z(\xi,y) = -B_z\delta(y-y_0)$$

$$\bar{\Phi}(\xi,y) = C_1\sin\eta_p y + C_2\cos\eta_p y - \frac{A}{2\eta_p}\sin\eta_p\left|y-y_0\right|$$
$$\bar{H}_z(\xi,y) = i\left(D_1\sin\eta_s y + D_2\cos\eta_s y - \frac{B_z}{2\eta_s}\sin\eta_s\left|y-y_0\right|\right)$$

$$\begin{bmatrix} (\xi^2-\eta_s^2)\cos\eta_p d & 2\xi\eta_s\cos\eta_s d \\ -2\xi\eta_p\sin\eta_p d & (\xi^2-\eta_s^2)\sin\eta_s d \end{bmatrix}\begin{bmatrix} C_2 \\ D_1 \end{bmatrix} = \begin{bmatrix} P_S \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} (\xi^2-\eta_s^2)\sin\eta_p d & -2\xi\eta_s\sin\eta_s d \\ 2\xi\eta_p\cos\eta_p & (\xi^2-\eta_s^2)\cos\eta_s d \end{bmatrix}\begin{bmatrix} C_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 0 \\ P_A \end{bmatrix}$$

$$P_S = \left((\xi^2-\eta_s^2)\frac{A}{2\eta_p}\sin\eta_p d_1 + 2\xi\eta_s\frac{B_z}{2\eta_s}\cos\eta_s d_1\right)$$

$$P_A = \left(2\xi\eta_p\frac{A}{2\eta_p}\cos\eta_p d_1 + (\xi^2-\eta_s^2)\frac{B_z}{2\eta_s}\sin\eta_s d_1\right)$$

Lamb wave part

$$\varepsilon_x = i\left(\sum_{j=0}^{j_S}\left[P_S(\xi_j^S)\frac{N_S(\xi_j^S)}{D_S'(\xi_j^S)}\right]e^{i(\xi_j^S x-\omega t)} + \sum_{j=0}^{j_A}\left[P_A(\xi_j^A)\frac{N_A(\xi_j^A)}{D_A'(\xi_j^A)}\right]e^{i(\xi_j^A x-\omega t)}\right)$$

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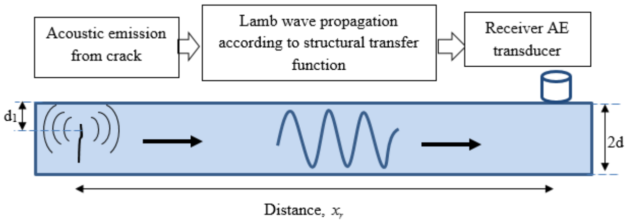
Inverse Algorithm

Forward problem



Excitation Potentials:

Time rate of excitation potential $\dot{A}^*(t)$ and $\dot{B}_z^*(t)(t)$

Integrate: Time profile of released energy $A^*(t)$ and $B_z^*(t)$

Calculate: Amplitude of the source

$$A(t)=\frac{h}{c_p^{\,2}}A^*(t)\,;\quad B_z(t)=\frac{h}{c_s^{\,2}}B_z^*(t)$$

⇩

Fourier transform of the signal

$$A(t)\rightarrow A(\omega)\,;\; B_z(t)\rightarrow B_z(\omega)$$

⇩

Calculate source term $P_S(\xi^S)$ and $P_A(\xi^A)$

⇩

Calculate structural transfer function

$$\frac{N_S(\xi^S)}{D_S'(\xi^S)}e^{i(\xi^S x_r)} \text{ and } \frac{N_A(\xi^A)}{D_A'(\xi^A)}e^{i(\xi^A x_r)}$$

⇩

Product of structural transfer functions and source term

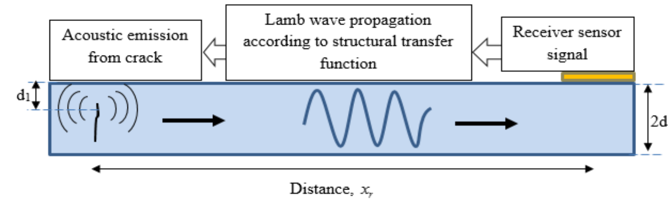
$$\varepsilon_x(\omega)=P_S(\xi^S,\omega)\frac{N_S(\xi^S)}{D_S'(\xi^S)}e^{i(\xi^S x_r)}+P_A(\xi^A,\omega)\frac{N_A(\xi^A)}{D_A'(\xi^A)}e^{i(\xi^A x_r)}$$

⇩

Perform inverse Fourier transform

$$\varepsilon_x(\omega)\rightarrow\varepsilon_x(t)$$

Inverse problem



Perform **Short time Fourier transforms (STFT)**

$$V_R(T)\rightarrow V_R(\omega)$$

⇩

Calculate structural transfer function

$$\frac{N_S(\xi^S)}{D_S'(\xi^S)}e^{i(\xi^S x_r)} \text{ and } \frac{N_A(\xi^A)}{D_A'(\xi^A)}e^{i(\xi^A x_r)}$$

⇩

Deconvolution of structural transfer functions and source term

$$P_S(\xi^S,\omega)=V_R^S(\omega)\Big/\frac{N_S(\xi^S)}{D_S'(\xi^S)}e^{i(\xi^S x_r)}$$

$$P_A(\xi^A,\omega)=V_R^A(\omega)\Big/\frac{N_A(\xi^A)}{D_A'(\xi^A)}e^{i(\xi^A x_r)}$$

⇩

Calculate source term $P_S(\xi^S)$ and $P_A(\xi^A)$

⇩

Calculate excitation source in frequency domain

$$A(\omega)\,;\; B_z(\omega)$$

⇩

Inverse Fourier transform of the excitation signal

$$A(\omega)\rightarrow A(t)\,;\; B_z(\omega)\rightarrow B_z(t)$$

⇩

Excitation Potentials:

Calculate: Excitation Potentials source

$$A^*(t)=c_p^{\,2}A(t)\,;\quad B_z^*(t)=c_s^{\,2}B_z(t)$$

Ref: Haider and Giurgiutiu, 2018

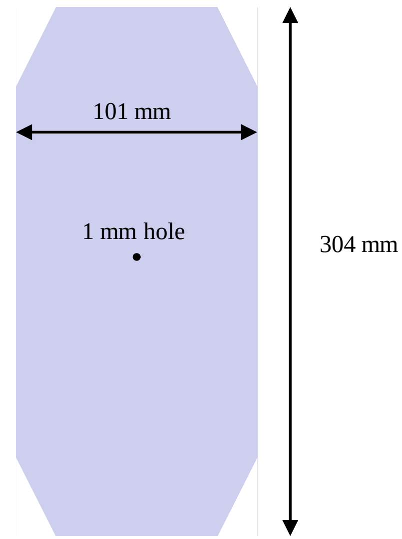
Experiments

Fabrication of Specimens

1 mm thick 304-stainless steel plate

12”

24”



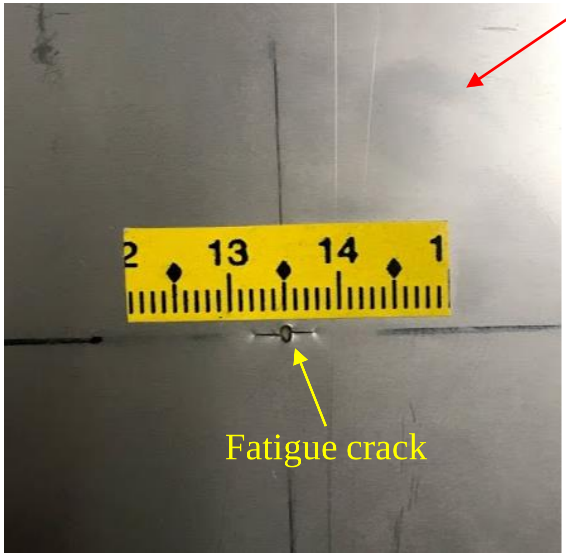
Test specimen

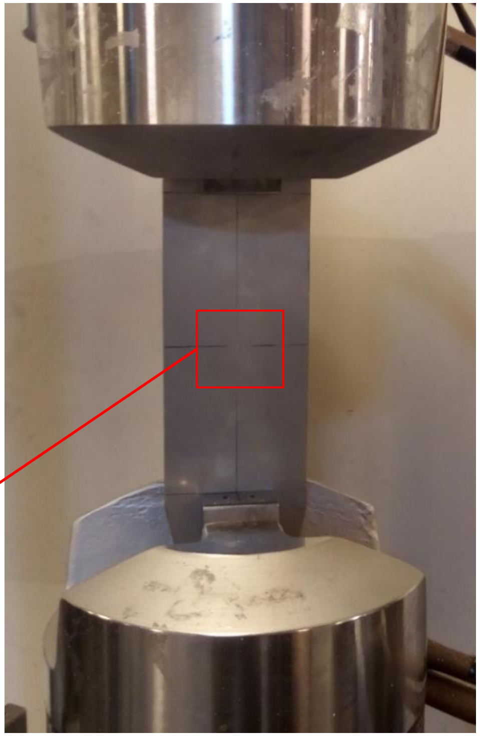
Tensile Strength, Ultimate: 505 MPa
Tensile strength, Yield: 215 MPa

- 304 mm X 101 mm coupons are fabricated
- 1 mm hole is provided at the geometric center for crack initiation

Fatigue Crack Generation in Steel Specimen

- Fatigue loading is applied with a minimum load of 2.5 kN and maximum load of 25 kN at 4 Hz
- At this load range the crack was initiated after 20000 cycles
- After 35,000 fatigue cycles we observed 6 mm crack





Fatigue test for crack generation

AE Measurement Experimental Setup

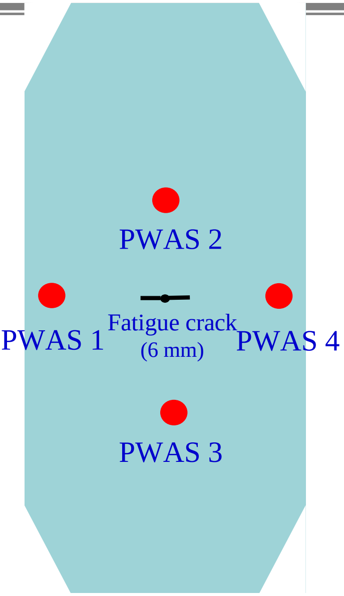
Test specimen mounted on

MTS machine

Acoustic pre-

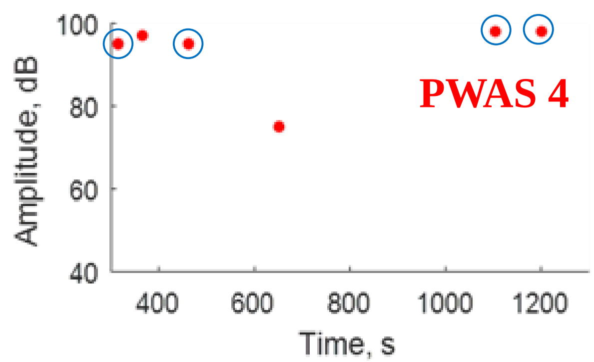
amplifier

Mistras AE instrument



Test specimen

- Loading: 1.8 kN- 18 kN
- Frequency: 0.5 Hz
- Fatigue loading was applied
- AE signals were collected using Mistras AE system
- Approximate crack growth during fatigue experiment from 6 mm to 10 mm (5000 cycles)

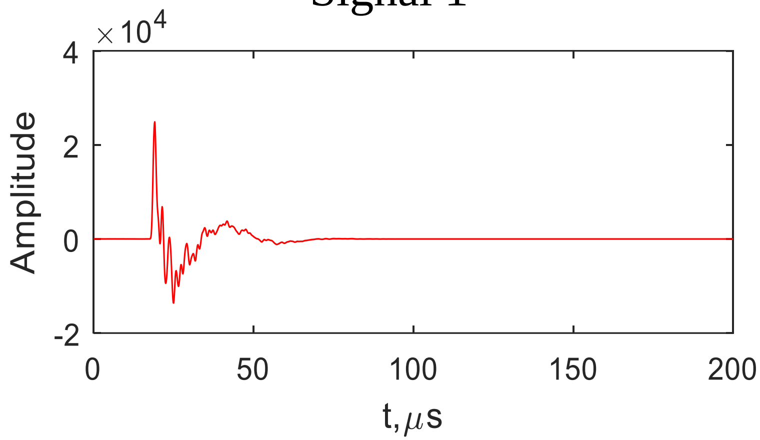


AE hits at PWAS 4 over 600 cycles

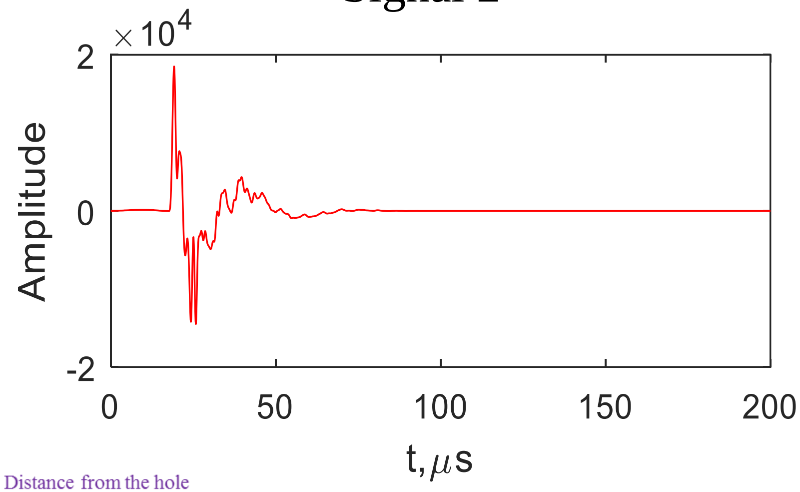
Results

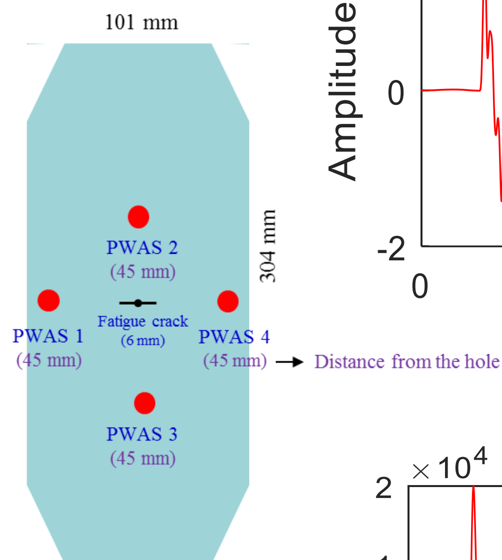
AE Signals Collected at PWAS 4

Signal 1

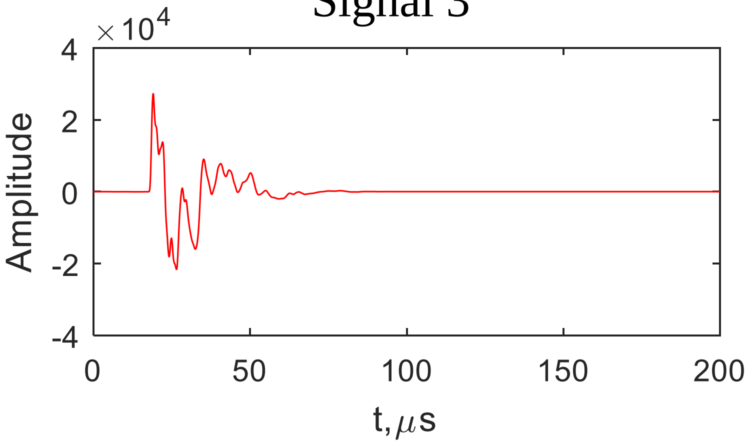


Signal 2

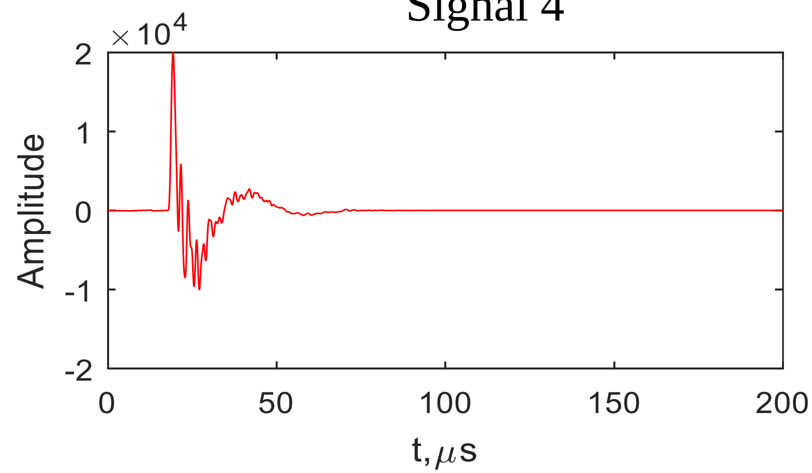




Signal 3



Signal 4



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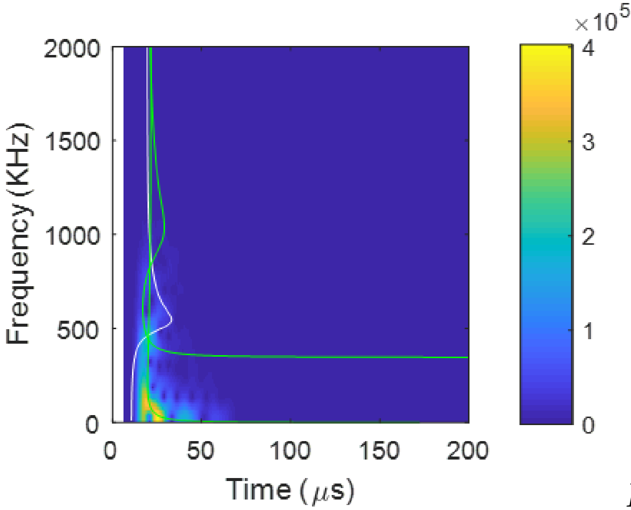
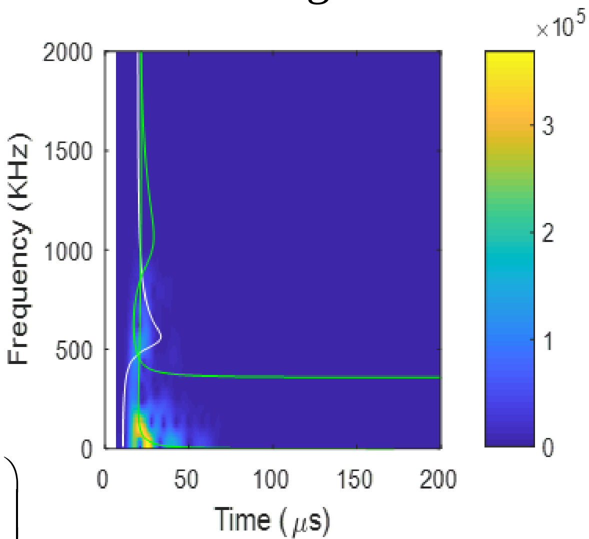
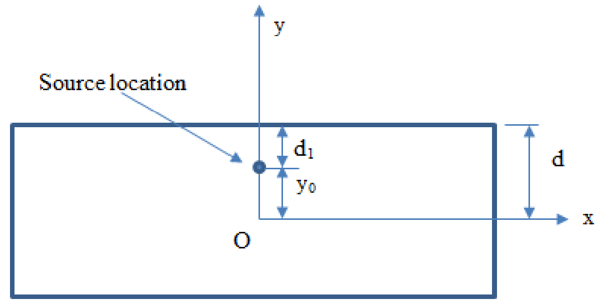
STFT of AE Signals

Signal 1



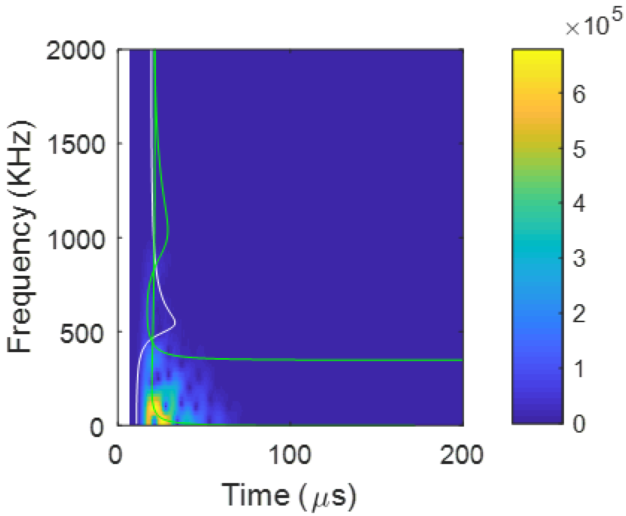
Signal 2



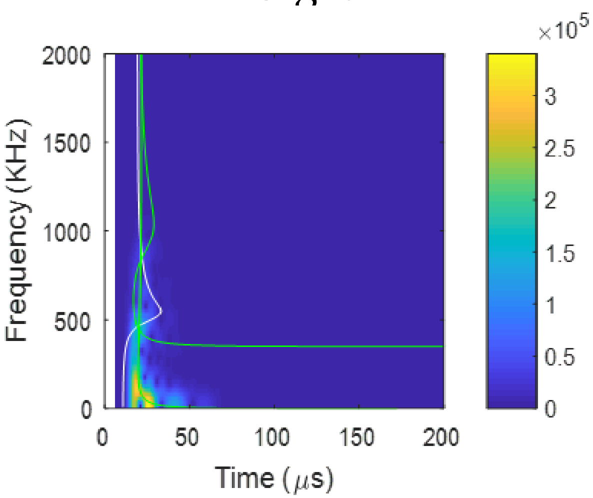


$$P_S = \left((\xi^2 - \eta_s^2) \frac{A}{2\eta_p} \sin \eta_p d_1 + 2\xi\eta_s \frac{B_z}{2\eta_s} \cos \eta_s d_1 \right)$$

$$P_A = \left(2\xi\eta_p \frac{A}{2\eta_p} \cos \eta_p d_1 + (\xi^2 - \eta_s^2) \frac{B_z}{2\eta_s} \sin \eta_s d_1 \right)$$

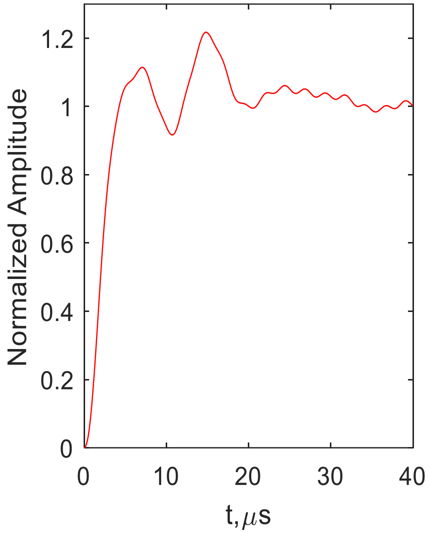


Signal 4



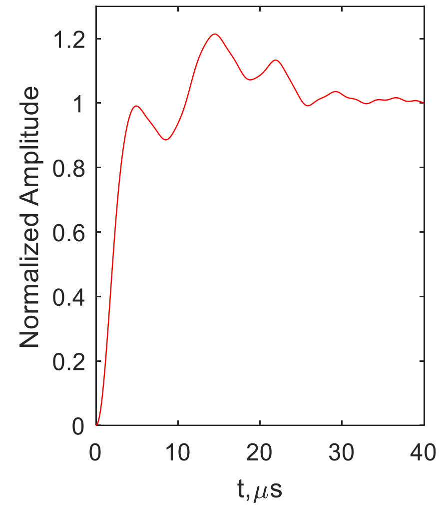
Pressure Excitation Potential Source of AE Signals

Signal 1

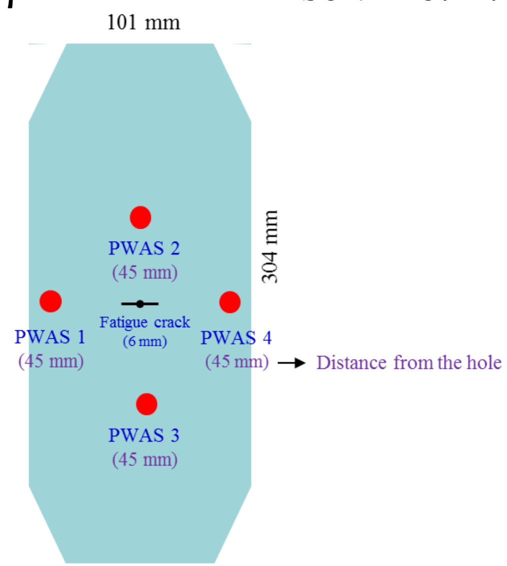


Rise time: 4.25 µs

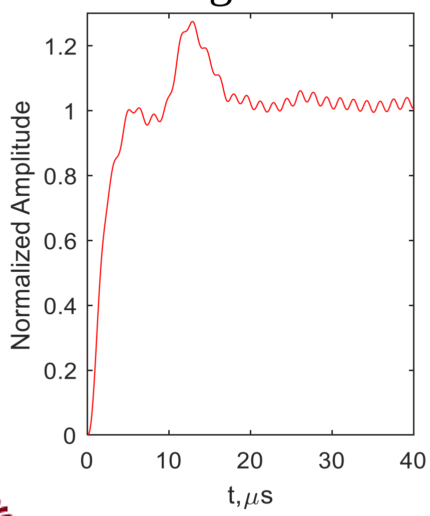
Signal 2



Rise time: 4.63 µs

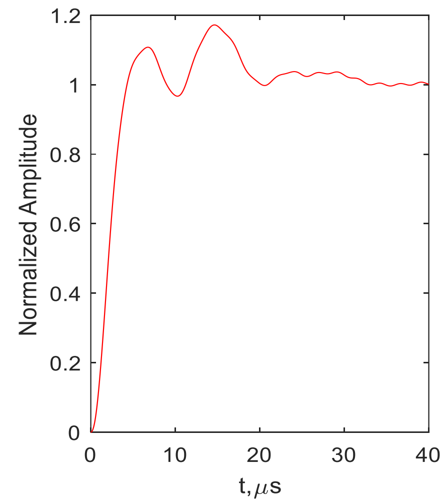


Signal 3



Rise time: 4.87 µs

Signal 4



Rise time: 4.26 µs

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Summary and Conclusions

- The guided waves generated by an acoustic emission (AE) event were analyzed through a Helmholtz potential approach
- The inhomogeneous elastodynamic Navier-Lame equation was expressed as a system of wave equations in terms of
  - Unknown scalar and vector potentials
  - Known scalar and vector excitation potentials
- An experiment was designed and performed to extract AE signals from a fatigue crack growth
- An inverse algorithm was developed to characterize the AE source during crack propagation
- The source characterization can provide information
  - About excitation potential from the fatigue crack
  - About a qualitative as well as quantitative description of the crack propagation phenomenon

Thanks